Benha University **Faculty of Engineering – Shoubra** **Eng. Mathematics & Physics Department** **Second Year Mech. Eng. – Production**		**Final Term Exam** **Date: January 9, 2017** **Course: Engineering Mathematics** **Code: EMP 291** **Duration: 3 hours**

- The Exam consists of **Two pages** Answer **All** Questions | **No. of questions: 3** **Total Mark: 80**

Question 1: **30**

1) ***Evaluate*** **the following integrals:**

$$\text{i}- \oint_C \left\{ \frac{\ln(5+z)}{(z+2)} \right\} dz \; ; \qquad C \text{ is } |z-1| = 4 .$$

$$\text{ii}- \oint_C \left\{ \frac{(4-3z)}{(z^2-3z-4)} \right\} dz \; ; \qquad C \text{ is } |z+1| = 2 .$$

2) **For the following data:**

x_i	1.00	1.25	1.50	1.75	2.00	2.25	2.50	2.75	3.00
$y_i = f(x_i)$	4.09	4.17	4.24	4.29	4.32	2.09	4.34	4.35	4.37

i- ***Find*** **the values** $f'(1)$, $f'(2.75)$, $f''(2.50)$.

ii- ***Evaluate*** **the following integral** ***Using*** **Simpson 's Rule:** $\int_{1.25}^{2.75} f(x)\,dx$.

3) ***Put*** **the following complex numbers in another formula:**

i) $z_1 = \{ \overline{(-3, \sqrt{3})} \}^{-15}$.   ii) $z_2 = (2\sqrt{3} + 2i)^{-1}$.

iii) $z_3 = \{ e^{[5 - i(29\pi/21)]} \}^{7}$.

4) **For the following data:**

$(1, 3), (2, 5), (3, 7), (4, 9), (5, 13)$.

i- ***Find*** **the least squares line that fits the data.**

ii- ***Using*** **the Newton 's Forward formula, Find the value of** $y = f(2.7)$.

LOOK ANOTHER PAGE

Question 2: 25

1) ***Expand*** of the function: $f(x) = x \; ; \quad x \in (0, \pi) \, , \quad f(x + 2\pi) = f(x) \, ,$ as ***Fourier*** cosine series.

2) ***Write*** the Fourier series, and ***Find*** Laplace 's transform $[L\{f(t)\}]$ for the following function: $f(x) = \cos^4(3x) \; ; \quad x \in [-\pi, \pi]$, and period 2π .

3) ***Find*** inverse Laplace 's transform $\left(L^{-1}\{F(s)\}\right)$ for the following functions:

i) $F(p) = \dfrac{(3p - p^3)^2}{p^{15}}$. ii) $F(p) = \dfrac{(p - 9)}{(p^2 - 36)}$. iii) $F(p) = \dfrac{5}{(p - 16)^9}$.

4) ***Find*** Laplace 's transform $[L\{f(t)\}]$ for the following functions:

i – $f(t) = 6t^4 - 7e^{-3t} \cdot \cos 5t$. ii – $f(t) = (3t^2 + 2t^3 + 5)^2$.

iii – $f(t) = (5t^8) \cdot (e^{-3t}) - (2t) \cdot (\sinh 2t)$.

Question 3: 25

1) ***Find*** the general solution for the following partial differential equations:

i) $u_{xx} - 6u_{xy} + 8u_{yy} = 13 e^{(5x + 3y)} - 9 \sin(7y)$.

ii) $u_x + 2u_y - 3u_x = 0 \; ; \qquad u(0, y) = 2e^{5y}$.

2) ***Evaluate*** the following functions :

i – $\Gamma(19/3) \div \Gamma(4/3)$. ii – $\Gamma(-7/6) \times \Gamma(23/6)$.

iii – $\beta(11/4, 25/4)$. iv – $\beta(\Gamma(3), \Gamma(4))$.

3) ***Find*** the value of $\sqrt[3]{12}$, by using the ***bisection*** method; where the number of iterations n , is 5 .

4) ***Solve*** the following differential equation, by ***Using*** the Taylor 's method:

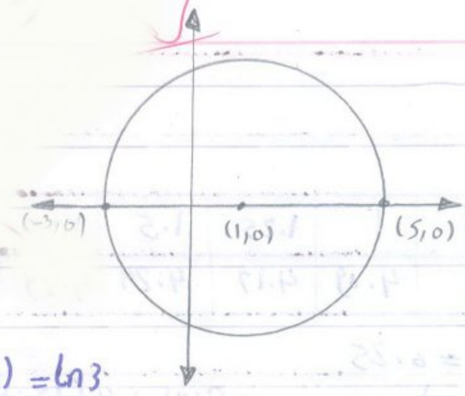
$$y'' + 4xy' - 7y = 0 \; ; \qquad y_0 = 2 \; , \qquad y'_0 = 0.3 \; .$$

ENDED TEST QUESTIONS

(1)

question (1) (1)

(i) $\oint_c \frac{\ln(5+z)}{z+2} \, .dz$ $\quad$ C is $|z-1| = 4$

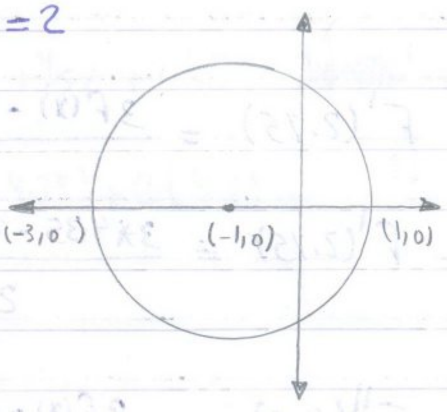


$z+2=0 \quad \therefore z_0 = -2 \in C$

$\therefore I = 2\pi i \, f(z_0) \quad ; \quad f(z_0) = \ln(5+z_0) = \ln 3$

$\therefore I = 2\pi i \ln 3$

(ii) $\oint_c \frac{4-3z}{(z^2-3z-4)} \, .dz \quad ; \quad$ C is $|z+1| = 2$



$z^2 - 3z - 4 = 0$

$(z+1)(z-4) = 0$

$\therefore z_1 = -1 \qquad \in C$

$\therefore z_2 = 4 \qquad \notin C$

$\therefore \oint_c \frac{4-3z}{z^2-3z-4} \, .dz = \oint \frac{\frac{(4-3z)}{(z-4)}}{z+1} \, .dz$

$\therefore I = 2\pi i \, f(z_0) \qquad \therefore f(z_0) = \frac{4-3*(-1)}{-1-4} = \frac{-7}{5}$

$\therefore I = \frac{-14}{5} \pi i$

2

(2)

(i)

x_i	1	1.25	1.5	1.75	2	2.25	2.5	2.75	3
y_i	4.09	4.17	4.24	4.29	4.32	2.09	4.34	4.35	4.3

$h = 0.25$

$$F'(1) = \frac{-3f(x) + 4f(x+h) - f(x+2h)}{2h} = \frac{-3f(1) + 4f(1.25) - f(1.5)}{2*0.25}$$

$$\therefore F'(1) = \frac{-3*4.09 + 4*4.17 - 4.24}{2*0.25} = 0.17$$

$$F'(2.75) = \frac{3f(x) - 4f(x-h) + f(x-2h)}{2h} = \frac{3f(2.75) - 4f(2.5) + f(2.25)}{2*0.25}$$

$$F'(2.75) = \frac{3*4.35 - 4*4.34 + 2.09}{2*0.25} = -4.44$$

$$F''(2.5) = \frac{2f(x) - 5f(x-h) + 4f(x-2h) - f(x-3h)}{h^2}$$

$$= \frac{2f(2.5) - 5f(2.25) + 4f(2) - f(1.75)}{0.25^2}$$

$$F''(2.5) = \frac{2*4.34 - 5*2.09 + 4*4.32 - 4.29}{(0.25)^2}$$

$$\boxed{F''(2.5) = 179.52}$$

Q, (3)

(ii)

*for simpson's rule

$$I = \frac{h}{3}\left[y_0 + y_n + 4(y_1 + y_3 + y_5) + 2(y_2 + y_4)\right]$$

$$I = \frac{0.25}{3}\left\{4.17 + 4.35 + 4(4.24 + 4.32 + 4.34) + 2(4.29 + 2.09)\right\}$$

$$I = 6.0733$$

Q1 4

(3)

(i) $Z_1 = (-3 - \sqrt{3}\, i)^{-15}$

$x = -3$, $y = -\sqrt{3}$ $\therefore r = \sqrt{(-3)^2 + (-\sqrt{3})^2} = 2\sqrt{3}$

$\theta = \tan^{-1}\frac{y}{x} = \frac{\tan^{-1}\sqrt{3}}{3} = \tan^{-1}\frac{\sqrt{3}}{3} = 30$ $(-,-)$ $\therefore \theta = 180 + 30 = 210^\circ$

$\theta = \frac{7\pi}{6}$

$\therefore Z_1 = (r e^{i\theta})^{-15} = (2\sqrt{3} \cdot e^{i\frac{7\pi}{6}})^{-15}$

$Z_1 = (2\sqrt{3})^{-15} \cdot e^{-i\frac{35}{2}\pi} = \frac{1}{(2\sqrt{3})^{15}} [\cancel{\cos\frac{35}{2}\pi} - i\sin\frac{35}{2}\pi]$

$\to 0$

$Z_1 = \frac{1}{(2\sqrt{3})^{15}} [\cos\frac{35}{2}\pi - i\sin\frac{35}{2}\pi]$

(ii) $Z_2 = (2\sqrt{3} + 2i)^{-1}$

$x = 2\sqrt{3}$, $y = 2$ $r = \sqrt{(2\sqrt{3})^2 + 2^2} = 4$

$\theta = \tan^{-1}\frac{1}{\sqrt{3}} = 30 = \frac{\pi}{6}$

$Z_2 = (re^{i\theta})^{-1} = (4e^{i\frac{\pi}{6}})^{-1} = \frac{1}{4(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6})}$

Q1

5

(iii) $Z_3 = \left[e^{[5 - i\frac{29}{21}\pi]}\right]^7 = e^{[7*5 - i\frac{29*7}{21}\pi]}$

$Z_3 = e^{[35 - i\frac{29}{3}\pi]} = e^{35} \cdot e^{-i\frac{29}{3}\pi}$

$Z_3 = e^{35}\left[\cos\frac{29}{3}\pi - i\sin\frac{29}{3}\pi\right]$

(4) (i)

The equation of line :-

$y = ax + b$

$$\sum_{i=1}^{5} y = a\sum_{i=1}^{5} x + b\sum_{i=1}^{5} 1 \quad (1)$$

$$\sum_{i=1}^{5} yx = a\sum_{i=1}^{5} x^2 + b\sum_{i=1}^{5} x \quad (2)$$

	x	y	x^2	xy
	1	3	1	3
	2	5	4	10
	3	7	9	21
	4	9	16	36
	5	13	25	65
Σ	15	37	55	135

Substi. at eq (1) and (2)

$37 = 15a + 5b \quad (3)$

$135 = 55a + 15b \quad (4)$

Apply eq (3) $*$ 3 ~~and~~

$111 = 45a + 15b \quad (5)$

$135 = 55a + 15b \quad (4)$

eq (4) − (5)

$24 = 10a \quad \therefore a = 2.4 \quad \therefore b = 0.2$

$\therefore$ The equation of line is $\boxed{y = 2.4x + 0.2}$

Q. 6

(ii)

$$y(x) = y_0 + (x-x_0)\frac{\Delta y_0}{1!h} + (x-x_0)(x-x_1)\frac{\Delta^2 y_0}{2!h} + (x-x_0)(x-x_1)(x-x_2)\frac{\Delta^3 y_0}{3!h}$$

$$+ (x-x_0)(x-x_1)(x-x_2)(x-x_3)\frac{\Delta^4 y_0}{4!h} + \cdots$$

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
1	3				
		2			
2	5		0		
		2		0	
3	7		0		2
		2		2	
4	9		2		
		4			
5	13				

$h = 1$

$x_0 = 1$, $y_0 = 3$, $\Delta y = 2$, $\Delta^2 y = 0$, $\Delta^3 y = 0$, $\Delta^4 y = 2$

$\therefore y(x) = 3 + (x-1) * 2 + (x-1)(x-2)(x-3)(x-4)\frac{2}{4! * 1}$

$\therefore f(2.7) = 6.438675$

Q2 7

question (2)

$f(x) = x \quad x\ (0, \pi) \quad , f(x+2\pi) = \cancel{f(x)}$

as fourier cosine series then $b_n = 0$

$L = \pi$

$$a_0 = \frac{2}{L}\int_0^L f(x)\,.\,dx = \frac{2}{\pi}\int_0^\pi x\,.\,dx = \frac{2}{\pi}\Big|_0^\pi \frac{x^2}{2}$$

$$a_0 = \frac{2}{\pi} * \frac{\pi^2}{2} = \pi$$

$$a_n = \frac{2}{L}\int_0^L f(x)\,.\cos\frac{n\pi x}{L} = \frac{2}{\pi}\int_0^\pi x\,.\cos\frac{n\pi x}{\pi}$$

$$\int_0^\pi x\,.\cos nx = \Big|_0^\pi \frac{x}{n}\sin nx - (1)\frac{(-\cos nx)}{n^2}\Big|$$

$$= \frac{x}{n}\sin n\pi - \frac{x}{n}\sin(0) + \frac{1}{n^2}\cos n\pi - \frac{1}{n^2}\cos(0)$$

$$= \frac{1}{n^2}(-1)^n - \frac{1}{n^2}(1) = \frac{1}{n^2}(-1)^{n+1}$$

$$\therefore S(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n\,.\cos\frac{n\pi x}{L}\,.$$

$$S(x) = \frac{\pi}{2} + \sum_{n=1}^{\infty}\frac{1}{n^2}(-1)^{n+1}\,.\cos nx$$

Q2 8

(2)

$f(x) = \cos^4(3x) \quad x \in [-\pi, \pi]$ The Period 2π

$\cos^4 3x = (\cos^2 3x)^2 = (\frac{1}{2} + \frac{1}{2}\cos 6x)^2 = \frac{1}{4} + \frac{1}{2}\cos 6x + \frac{1}{4}\cos^2 6x$

$= \frac{1}{4} + \frac{1}{2}\cos 6x + \frac{1}{4}(\frac{1}{2} + \frac{1}{2}\cos 12x) = \frac{1}{4} + \frac{1}{2}\cos 6x + \frac{1}{8} + \frac{1}{8}\cos 12x$

$= \frac{3}{8} + \frac{1}{2}\cos 6x + \frac{1}{8}\cos 12x$

all fourier Coffecient equal Zero and $2a_0 = \frac{3}{8}$, $a_6 = \frac{1}{2}$ $a_8 = \frac{1}{8}$

$a_0 = \frac{3}{4}$, $a_6 = \frac{1}{2}$ $a_8 = \frac{1}{8}$

$S(x) = \frac{3}{8} + \sum_{n=1}^{\infty} \frac{1}{2} \cos \frac{n\pi x}{2\pi} + \frac{1}{8} \cos \frac{n\pi x}{2\pi}$

(ii) * to find LaPlace's trans form

$f(t) = \frac{3}{8} + \frac{1}{2}\cos 6t + \frac{1}{8}\cos 12t$

$f(s) = \frac{3}{8}\cancel{s}\,\frac{}{s} + \frac{s}{2(s^2+36)} + \frac{s}{8(s^2+144)}$

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2 (9)

(i) $f(p) = \frac{(3p - p^3)^2}{p^{15}} = \frac{9p^2 - 6p^4 + p^6}{p^{15}} = \frac{9}{p^{13}} - \frac{6}{p^{11}} + \frac{1}{p^9}$

$\frac{9}{p^{13}} \longrightarrow n+1 = 13 \quad \therefore n = 12$

$t^{12} = \frac{\Gamma(13)}{p^{13}} \quad \therefore \frac{9}{p^{13}} = \frac{\Gamma(13)}{9} t^{12} = t^{12}\ \frac{9}{12!}$

$\frac{6}{p^{11}} = t^{10}\ \frac{6}{10!}$

$\frac{1}{p^9} = \frac{1}{\Gamma(9)} t^8$

$\therefore f(p) = \frac{9}{12!} t^{12} - \frac{6}{10!} t^{10} + \frac{1}{8!} t^8$

2

(ii) $f(p) = \frac{p-9}{p^2 - 36}$

(iii) $f(p) = \frac{5}{(p-16)^9}$

$t^n = \frac{\Gamma(n+1)}{S^{n+1}} \cdot e^{at} \quad a = 16$

$n+1 = 9 \quad n = 8$

$\frac{5}{8!} t^8 \cdot e^{at} = \frac{5}{(p-16)^9}$

$f(p) = \frac{5}{8!} t^8 \cdot e^{16t}$

$e^{at} \cdot t^n = \frac{\Gamma(n+1)}{(S-a)^{n+1}}$

Q2 10

4

(i) $f(t) = 6t^4 - 7e^{-3t} \cdot \cos 5t$

$$f(s) = \frac{6\Gamma(5)}{s^5} - \frac{7(s+3)}{((s+3)^2+5^2)} = \frac{720}{s^5} - \frac{7(s+3)}{((s+3)^2+25)}$$

(ii) $f(t) = (3t^2 + 2t^3 + 5)^2$

$$f(s) = \left(\frac{3\Gamma(3)}{s^3} + \frac{2\Gamma(4)}{s^4} + 5\right)^2$$

(iii) $f(t) = 5t^8 \cdot e^{-3t} - 2t \sinh 2t$

$5t^8 \to f(s) = \frac{5\Gamma(9)}{s^9}$ $\quad \therefore 5t^8 \cdot e^{-3t} = \frac{5\Gamma(9)}{(s+3)^9}$

$2t \sinh 2t \to n = 1$

$f(s) = (-1)^n F^n(s)$

$\sinh 2t \to f(s) = \frac{2}{s^2-4}$

$\therefore f(s) = 2(-1) \frac{d}{ds}\left(2(s^2-4)^{-1}\right)$

$= -2 * -2(2s)(s^2-4)^{-2} = \frac{8s}{(s^2-4)^{+2}}$

$$\therefore f(s) = \frac{5(8!)}{(s+3)^9} - \frac{8s}{(s^2-4)^2}$$

(11) Q3

question (3)

① (1)

$$u_{xx} - 6u_{xy} + 8u_{yy} = 13\, e^{(5x+3y)} - 9\sin(7y)$$

$$\cancel{D^2 - 6D}\, u_c = k^2 - 6k + 8 \qquad \therefore k_1 = 4 \quad k_2 = 2$$

$$\therefore u_c = f_1(y+4x) + f_2(y+2x)$$

$$u_I = \frac{13}{D^2 - 6DE + 8E^2} e^{5x+3y} - \frac{9}{D^2 - 6DE + 8E^2} \sin 7y$$

$$u_I = \frac{13}{7} e^{5x+3y} - \frac{9}{392} \sin 7y$$

$$\therefore u = u_I + u_c$$

$$u = f_1(y+4x) + f_2(y+2x) + \frac{13}{7} e^{5x+3y} - \frac{9}{392} \sin 7y$$

Q3

12

(ii) $u_x + 2u_y - 3u = 0$ $\quad u(0,y) = 2e^{5y}$

$u(x,y) = e^{a\cancel{x}+by}$

$u_x = a\, e^{ax+by} = au$

$u_y = b\, e^{ax+by} = bu$

$\therefore au + 2bu - 3u = 0 \qquad u(a + 2b - 3) = 0 \qquad u \neq 0$

$\boxed{a = 3 - 2b}$

$\therefore u(x,y) = e^{(3-2b)x + by}$

$\therefore e^{(3-2b)x + by} = 2e^{5y}$

$\therefore by = 5y$ when the cofficient of $u(x,y) = 2$

$\therefore b = 5 \qquad \therefore a = 3 - 10 = -7$

$\therefore u(x,y) = 2e^{-7x + 5y}$

Q3 (13)

(i) $\dfrac{\Gamma(19/3)}{\Gamma(4/3)} = = \dfrac{(2)\ 19/3 * 16/3 * 13/3 * 10/3 * 7/3 * \cancel{\Gamma 4/3} * \frac{7}{3}}{\Gamma(\cancel{4/3})}$

$= \cancel{1338.98}\ 2656.3$

(ii) $\Gamma\left(\frac{-7}{6}\right) * \Gamma\left(\frac{28}{6}\right)$

(iii)

$\beta\left(\frac{11}{4}, \frac{25}{4}\right) = \dfrac{\Gamma(11/4)\cdot\Gamma 25/4}{\Gamma\left(\frac{11}{4}+\frac{25}{4}\right)} = \dfrac{\Gamma(11/4)\cdot\Gamma(25/4)}{\Gamma 177/28}$

$= \dfrac{7/4 * 3/4\ \Gamma 3/4 * \frac{25}{4} * 21/4 * 17/4 * 13/4 * 9/4 * 5/4\ \Gamma 5/4}{\Gamma\ 177/28}$

$= \dfrac{(7/4)(3/4)(25/4)(21/4)(17/4)(13/4)(9/4)(5/4)(1/4)\ \Gamma(3/4)\cdot\Gamma(1/4)}{(149/28)(121/28)(93/28)(65/28)(37/28)(9/28)\ \Gamma\ 9/28}$

$= 7.404 * \dfrac{\Gamma(3/4)(\Gamma 1/4)}{\Gamma 9/28} = 7.404\ \dfrac{\pi}{\Gamma 9/28\ \sin\frac{25}{4}\pi}$

7

(14)

(iv) $\beta(\Gamma(3), \Gamma(4)) = \beta(2, 6) = \dfrac{\Gamma(2)\cdot\Gamma(6)}{\Gamma 8}$

$= \dfrac{1 * 5!}{7!} = \dfrac{1}{42}$

(3)

n	a	b	xn	yn
0				
1				
2				
3				
4				
5				

15

(4)

$y'' + 4xy' - 7y = 0 \quad y(0) = 2 \quad , y'_0 = 0.3$

$a = 0 \quad , b = 2$

$$y(x) = y(a) + (x-a)\, y'(a) + \frac{(x-a)^2}{2!}(y''(a)) + \frac{(x-a)^3}{3!}\, y'''(a)$$

$y'_0 = 0.3$

$y''(0) = 7y - 4xy'$

$= 7*2 - 4*0*0.3 = 14$

$$y''' = 7\frac{dy}{dx} - 4y' - 4xy''$$

$y'''(0) = 7*0.3 - 4*0.3 - 4*0*14 = 0.9$

$$y(x) = 2 + 0.3(x) + 7(x^2) + \frac{3}{20}x^3$$

Q2 (8)

(2)

$f(x) = \cos^4(3x) \quad x \in [-\pi, \pi]$ The Period 2π

$\cos^4 3x = (\cos^2 3x)^2 = \left(\frac{1}{2} + \frac{1}{2}\cos 6x\right)^2 = \frac{1}{4} + \frac{1}{2}\cos 6x + \frac{1}{4}\cos^2 6x$

$= \frac{1}{4} + \frac{1}{2}\cos 6x + \frac{1}{4}\left(\frac{1}{2} + \frac{1}{2}\cos 12x\right) = \frac{1}{4} + \frac{1}{2}\cos 6x + \frac{1}{8} + \frac{1}{8}\cos 12x$

$= \frac{3}{8} + \frac{1}{2}\cos 6x + \frac{1}{8}\cos 12x$

all fourier coffecient equal zero and $2a_0 = \frac{3}{8}$, $a_6 = \frac{1}{2}$, $a_8 = \frac{1}{8}$

$a_0 = \frac{3}{4}$, $a_6 = \frac{1}{2}$, $a_8 = \frac{1}{8}$

$S(x) = \frac{3}{8} + \sum_{n=1}^{\infty} \frac{1}{2} \cos \frac{n\pi x}{2\pi} + \frac{1}{8} \cos \frac{n\pi x}{2\pi}$

ii) * to find laplace's transform

$f(t) = \frac{3}{8} + \frac{1}{2}\cos 6t + \frac{1}{8}\cos 12t$

$f(s) = \frac{3}{8\cancel{s}\,s} + \frac{s}{2(s^2+36)} + \frac{s}{8(s^2+144)}$